第2章 线性电阻电路的等效变换

重点

1. 电路等效的概念
2. 电阻的串、并联
3. Y—Δ 变换
4. 实际电源的等效变换
5. 输入电阻

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2.1 引言

线性直流电路是最简单、最基本的一类电路

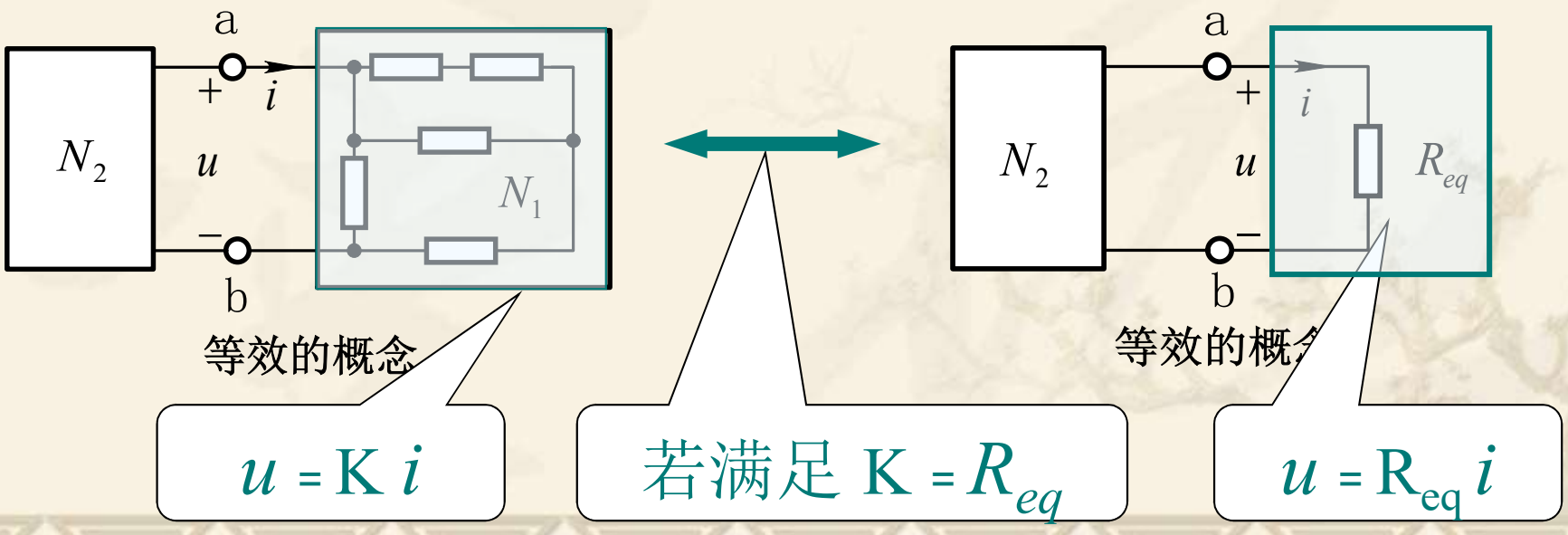
- 线性电路：线性元件和电源组成
- 直流电路：激励为直流电源，响应（电压、电流）也都是直流量（即恒定量）
- 电路的基本计算方法：

  等效变换法
  列方程法
  电路定理法

线性直流电路分析方法广泛应用或推广用于后续各章

2.2 电路的等效变换

- 等效电路：两个电路具有相同的端口特性
  （即端口电压、电流关系方程相同）
- 等效变换：两个端口特性相同的电路互换后，
  对外等效（即不影响外电路的响应）。

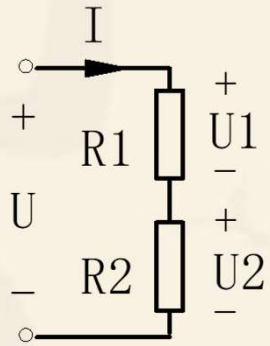


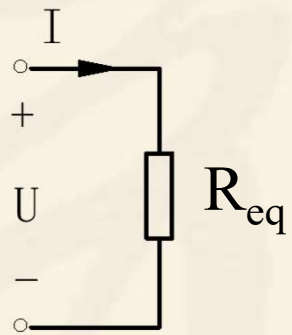
2.3 电阻的串联与并联

- 串联：各元件依次连接，流过同一电流
- 并联：各元件接于同一对节点之间，承受同一电压

一、电阻的串联

1、 等效电阻





$$U = U_1 + U_2 = R_1 I + R_2 I = (R_1 + R_2) I = R_{eq} I$$

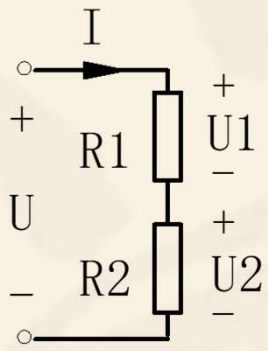
$$R_{eq} = R_1 + R_2 \xrightarrow{\text{推广}} R_{eq} = \sum_{k=1}^{n} R_k$$

2.3 电阻的串联与并联

2. 分压作用：

$$U_1 = \frac{U}{R_1 + R_2} R_1$$

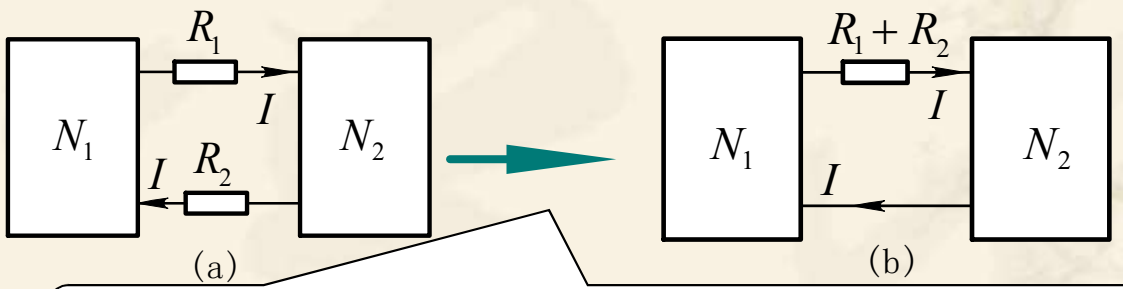
$$U_2 = \frac{R_2}{R_1 + R_2} U$$



3. 功率分配：

$$\frac{P_1}{P_2}\left(=\frac{I^2 R_1}{I^2 R_2}\right) = \frac{R_1}{R_2}$$

思考：对外等效



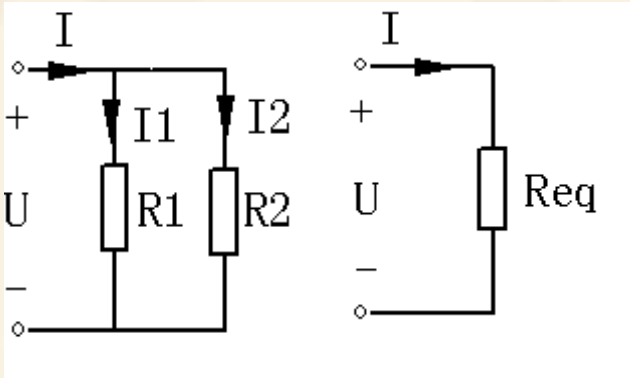
N1节点与N2节点间电压改变了

注：如此等效之后，电路中的那些量发生了变化？

2.3 电阻的串联与并联

一、电阻的并联

1、 等效电阻



$$I = I_1 + I_2 = \frac{U}{R_1} + \frac{U}{R_2} = (G_1 + G_2)U = G_{eq}U$$

$$G_{eq} = G_1 + G_2 \quad 或 \quad R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$G_{eq} = G_1 + G_2 \xrightarrow{推广} G_{eq} = \sum_{k=1}^{n} G_k$$

2. 分流作用：

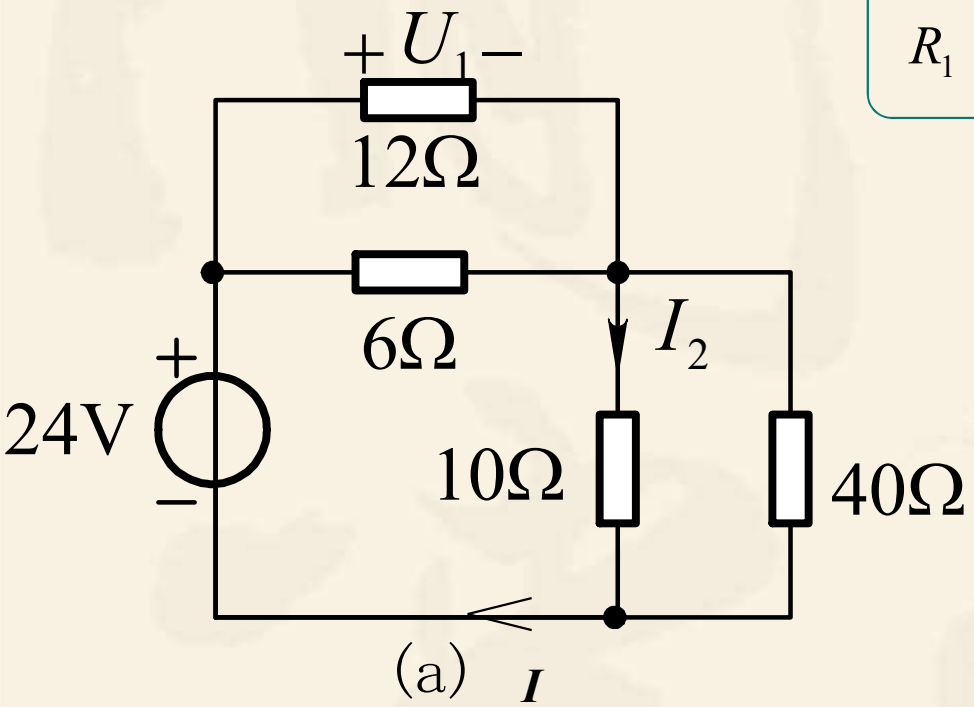
$$I_1 = \frac{R_2}{R_1 + R_2} I \qquad I_2 = \frac{R_1}{R_1 + R_2} I$$

3. 功率的分配

$$\frac{P_1}{P_2} \left(= \frac{U^2 G_1}{U^2 G_2}\right) = \frac{G_1}{G_2}$$

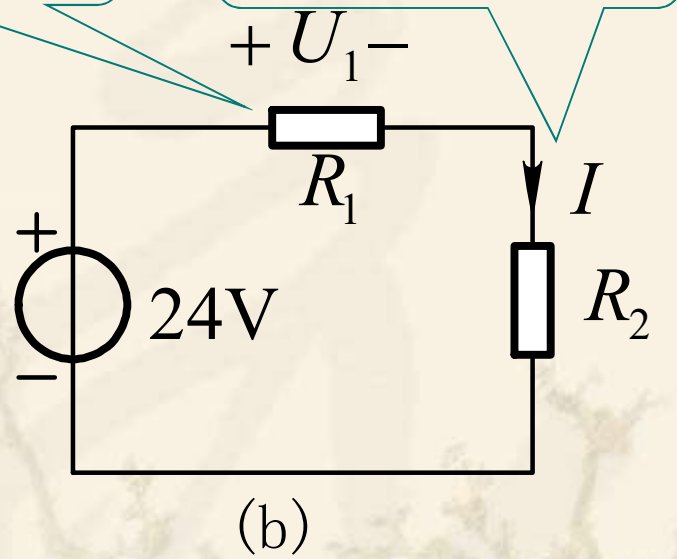
2.3 电阻的串联与并联

例题2.1：求电压U_1和电流I_2



$$R_1 = \frac{6\times 12}{6+12} = 4\Omega$$

$$R_2 = \frac{10\times 40}{10+40} = 8\Omega$$

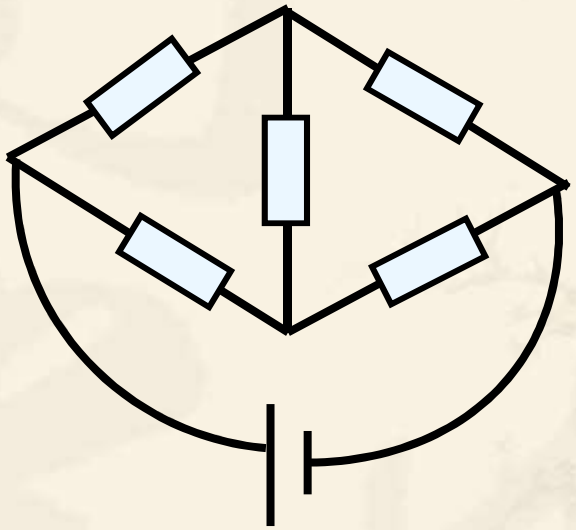


$$U_1 = \frac{R_1}{R_1+R_2}\times 24 = 8V \qquad I = \frac{24}{R_1+R_2} = 2A \qquad I_2 = \frac{40}{10+40}\times I = 1.6A$$

2.4 电阻的星形和三角形联接

- 问题的引出：

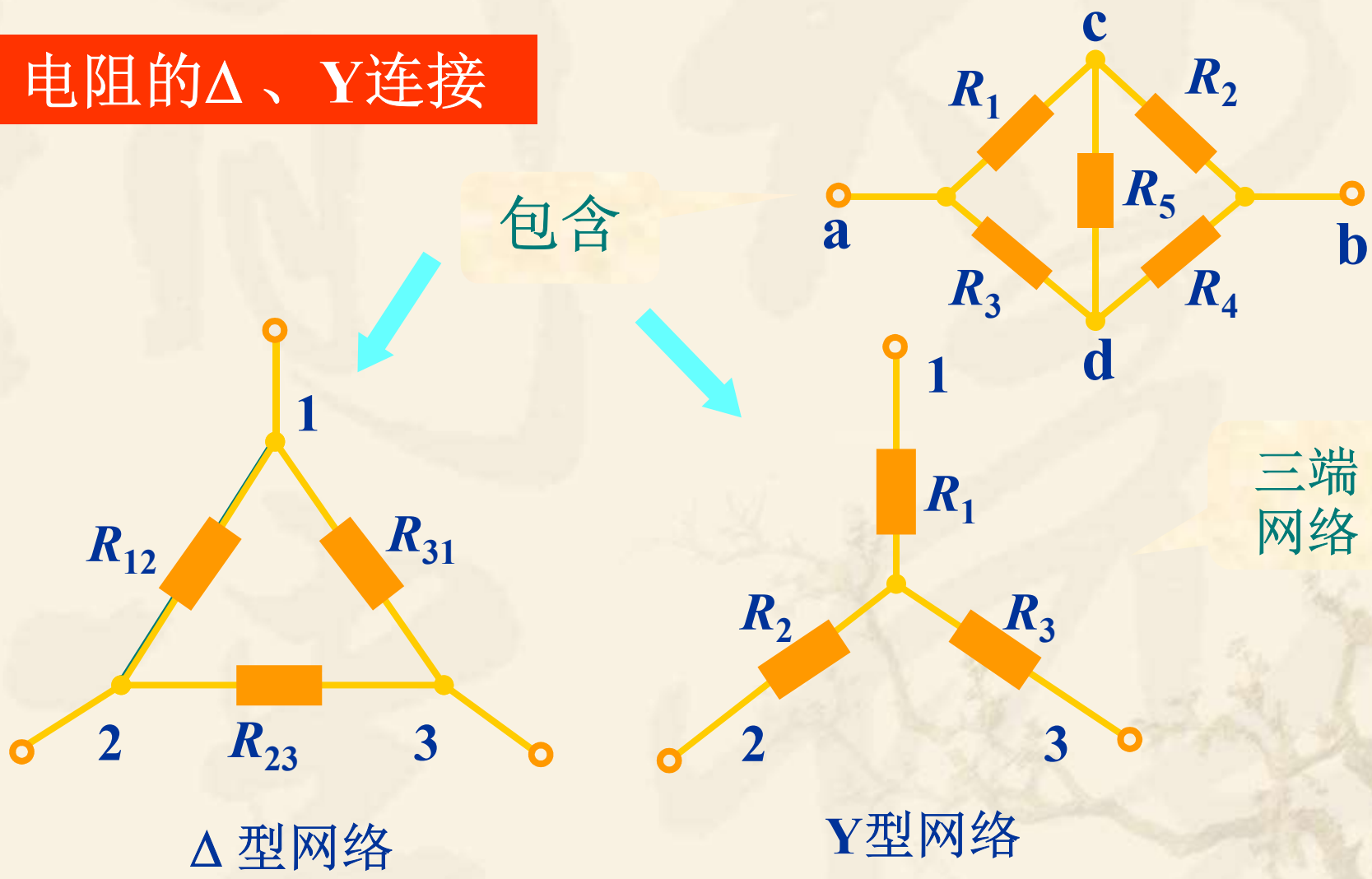
  电桥电路中的电阻连接关系既不是并联，也不是串联，所以其等效电路不能用电阻的串、并联等效变换



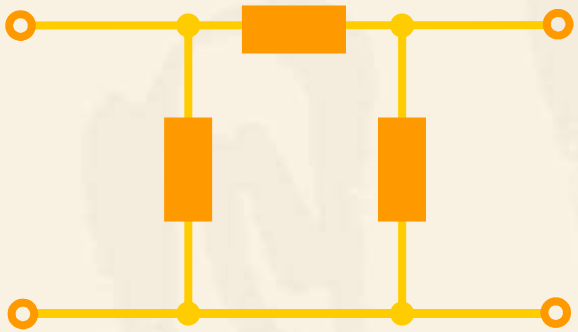
- 电桥电路结构特点：星形或三角形结构

2.4 电阻的Y形连接和Δ形连接的等效变换

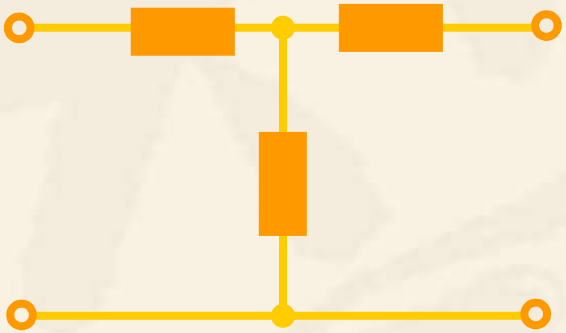
1. 电阻的Δ、Y连接



Δ，Y 网络的变形



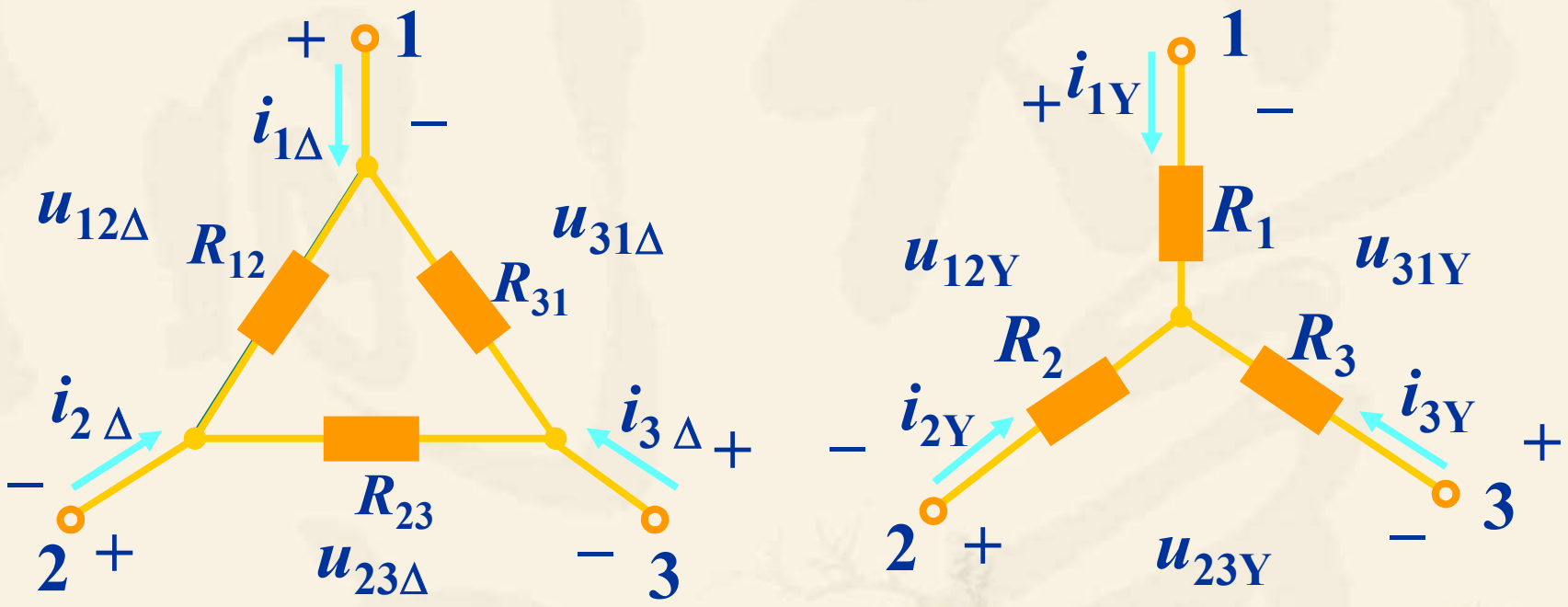
π 型电路 (Δ 型)



T 型电路 (Y、星型)

这两个电路当它们的电阻满足一定的关系时，能够相互等效

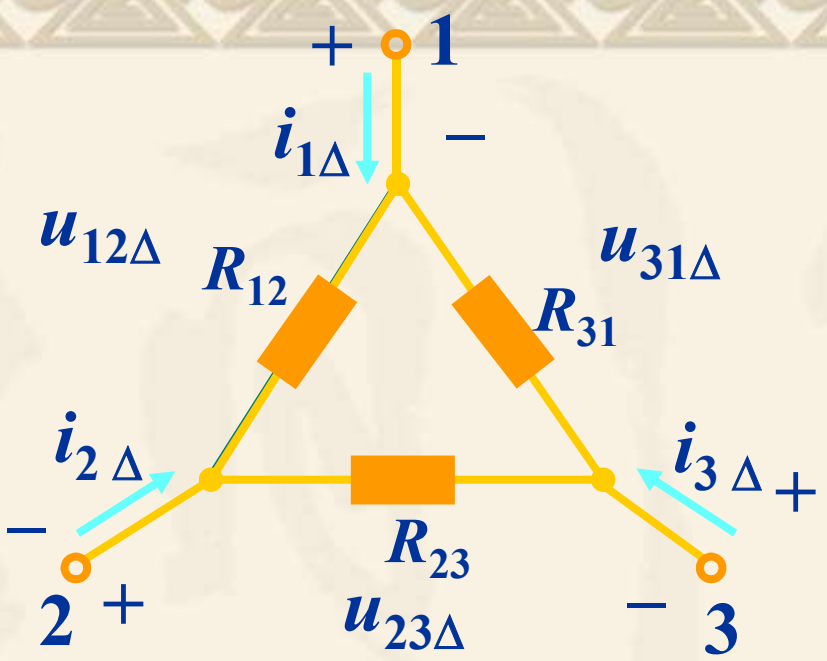
2. Δ—Y 变换的等效条件

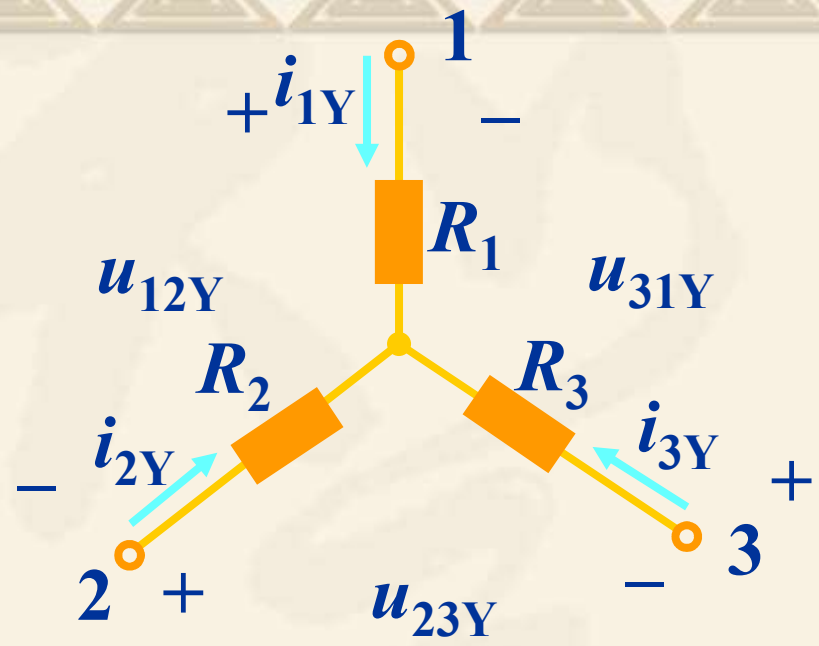


等效条件：

$$i_{1\Delta}=i_{1Y}, \quad i_{2\Delta}=i_{2Y}, \quad i_{3\Delta}=i_{3Y},$$

$$u_{12\Delta}=u_{12Y}, \quad u_{23\Delta}=u_{23Y}, \quad u_{31\Delta}=u_{31Y}$$





Δ接: 用电压表示电流

$$\left.\begin{aligned} i_{1\Delta} &= u_{12\Delta}/R_{12} - u_{31\Delta}/R_{31} \\ i_{2\Delta} &= u_{23\Delta}/R_{23} - u_{12\Delta}/R_{12} \\ i_{3\Delta} &= u_{31\Delta}/R_{31} - u_{23\Delta}/R_{23} \end{aligned}\right\} \quad (1)$$

Y接: 用电流表示电压

$$\left.\begin{aligned} u_{12Y} &= R_1 i_{1Y} - R_2 i_{2Y} \\ u_{23Y} &= R_2 i_{2Y} - R_3 i_{3Y} \\ u_{31Y} &= R_3 i_{3Y} - R_1 i_{1Y} \\ i_{1Y} + i_{2Y} + i_{3Y} &= 0 \end{aligned}\right\} \quad (2)$$

由式(2)解得：

$$\left.\begin{aligned} i_{1Y} &= \frac{u_{12Y}R_3 - u_{31Y}R_2}{R_1R_2 + R_2R_3 + R_3R_1} \\ i_{2Y} &= \frac{u_{23Y}R_1 - u_{12Y}R_3}{R_1R_2 + R_2R_3 + R_3R_1} \\ i_{3Y} &= \frac{u_{31Y}R_2 - u_{23Y}R_1}{R_1R_2 + R_2R_3 + R_3R_1} \end{aligned}\right\} (3)$$

$$\left.\begin{aligned} i_{1\Delta} &= u_{12\Delta}/R_{12} - u_{31\Delta}/R_{31} \\ i_{2\Delta} &= u_{23\Delta}/R_{23} - u_{12\Delta}/R_{12} \\ i_{3\Delta} &= u_{31\Delta}/R_{31} - u_{23\Delta}/R_{23} \end{aligned}\right\} (1)$$

根据等效条件，比较式(3)与式(1)，得Y型→Δ型的变换条件：

$$\left.\begin{aligned} R_{12} &= R_1 + R_2 + \frac{R_1R_2}{R_3} \\ R_{23} &= R_2 + R_3 + \frac{R_2R_3}{R_1} \\ R_{31} &= R_3 + R_1 + \frac{R_3R_1}{R_2} \end{aligned}\right\}$$

或

$$\left.\begin{aligned} G_{12} &= \frac{G_1G_2}{G_1 + G_2 + G_3} \\ G_{23} &= \frac{G_2G_3}{G_1 + G_2 + G_3} \\ G_{31} &= \frac{G_3G_1}{G_1 + G_2 + G_3} \end{aligned}\right\}$$

类似可得到由Δ型→Y型的变换条件：

$$\left.\begin{aligned} G_1 &= G_{12} + G_{31} + \frac{G_{12}G_{31}}{G_{23}} \\ G_2 &= G_{23} + G_{12} + \frac{G_{23}G_{12}}{G_{31}} \\ G_3 &= G_{31} + G_{23} + \frac{G_{31}G_{23}}{G_{12}} \end{aligned}\right\} \text{或} \left.\begin{aligned} R_1 &= \frac{R_{12}R_{31}}{R_{12}+R_{23}+R_{31}} \\ R_2 &= \frac{R_{23}R_{12}}{R_{12}+R_{23}+R_{31}} \\ R_3 &= \frac{R_{31}R_{23}}{R_{12}+R_{23}+R_{31}} \end{aligned}\right\}$$

简记方法：

$$R_Y = \frac{\Delta\text{相邻电阻乘积}}{\sum R_\Delta} \quad \text{或} \quad G_\Delta = \frac{\text{Y相邻电导乘积}}{\sum G_Y}$$

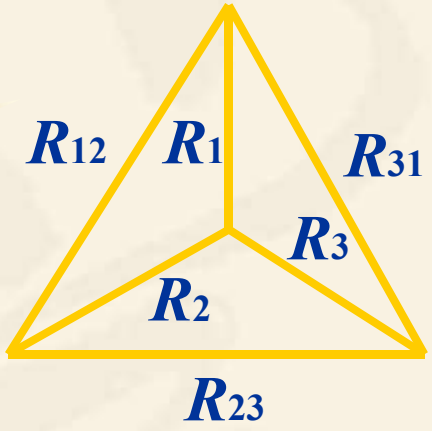
Δ变Y

Y变Δ

特例：若三个电阻相等(对称)，则有

$$R_{\Delta} = 3R_{Y}$$

外大内小



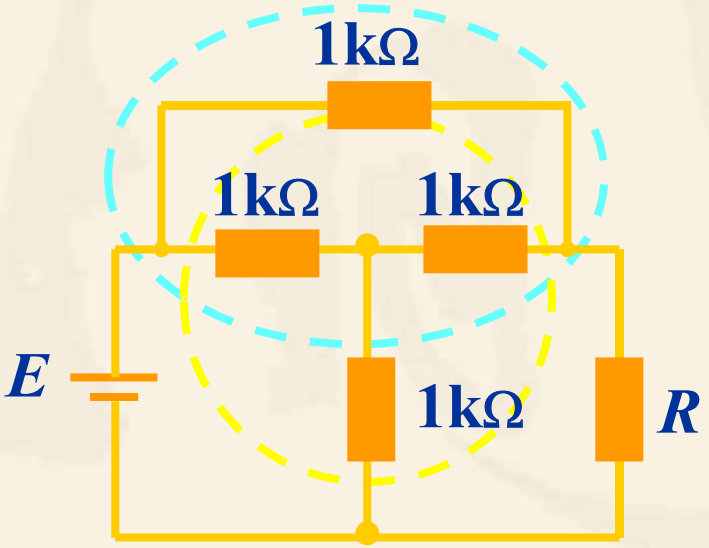
注意

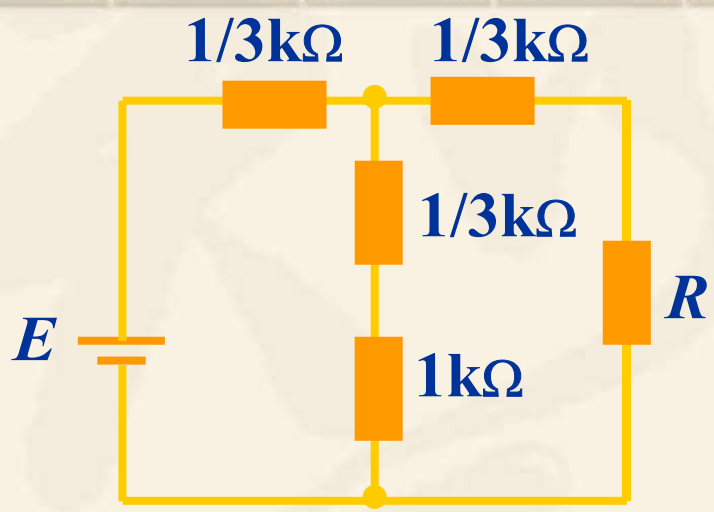
(1) 等效对外部(端钮以外)有效，对内不成立；

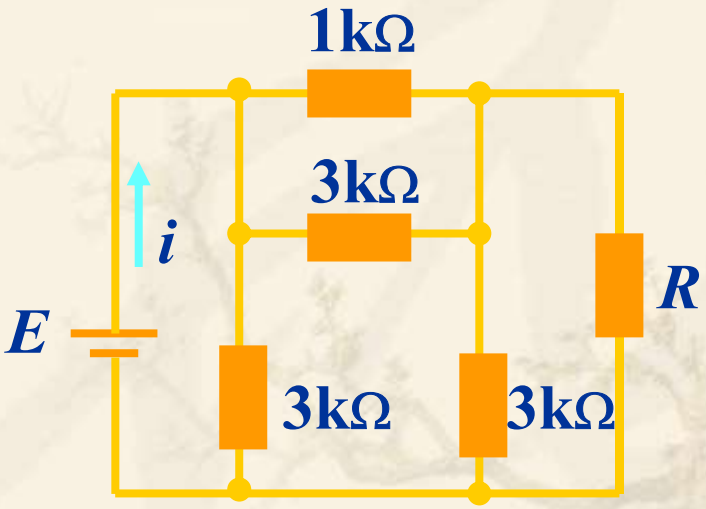
(2) 等效电路与外部电路无关；

(3) 用于简化电路。

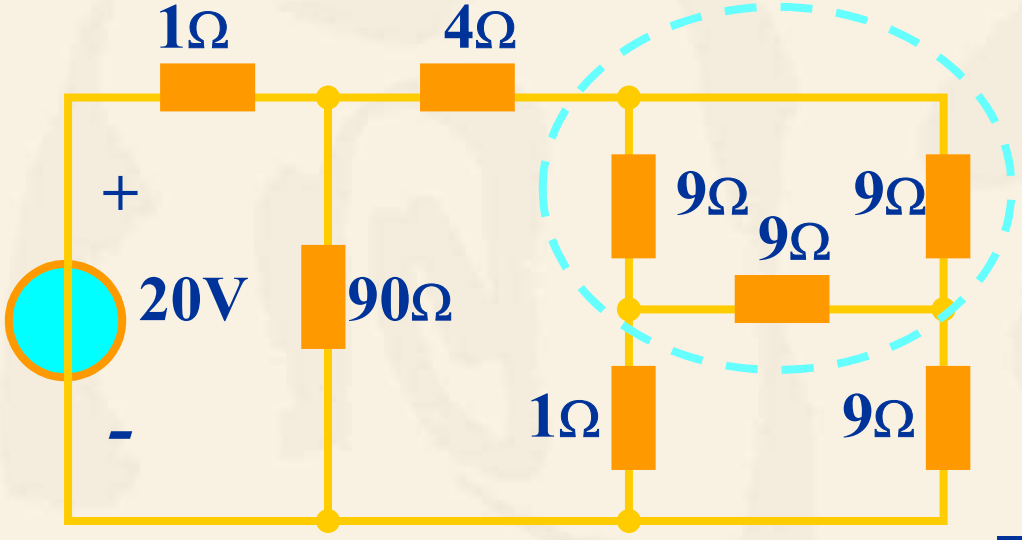
例 桥 T 电路

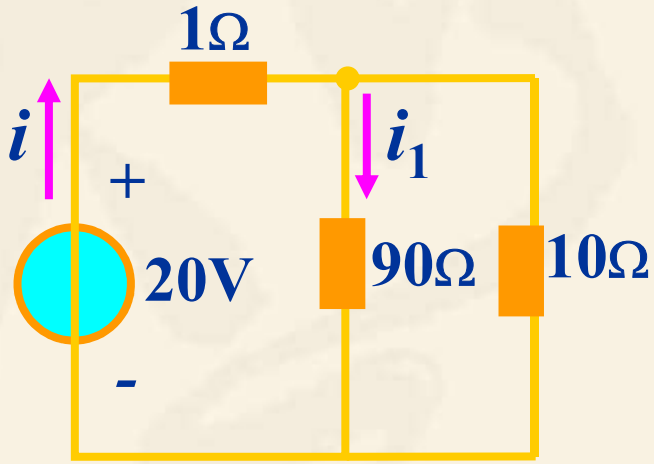


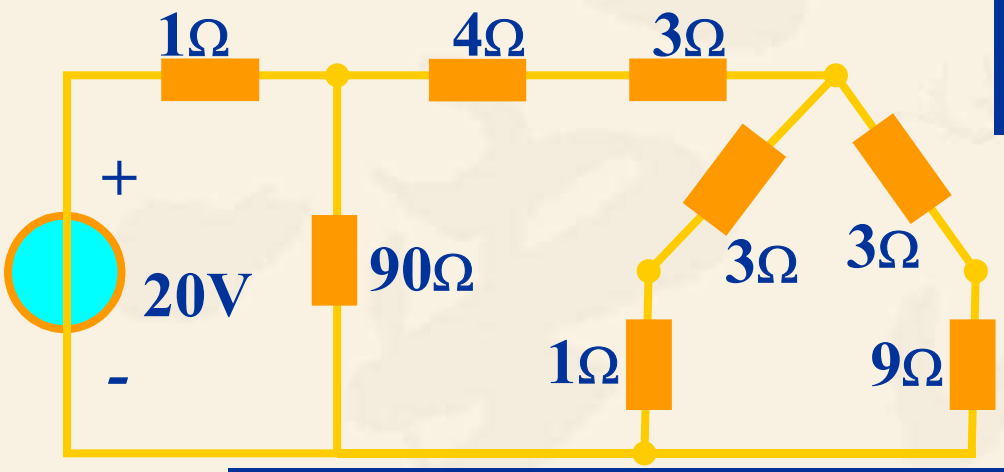




例 计算90Ω电阻吸收的功率







$$R_{eq} = 1 + \frac{10 \times 90}{10 + 90} = 10\Omega$$

$$i = 20/10 = 2A$$

$$i_1 = \frac{10 \times 2}{10 + 90} = 0.2A$$

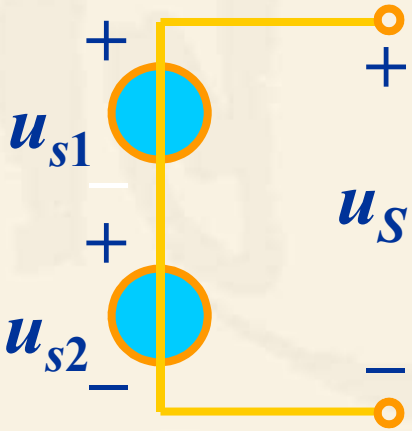
$$P = 90 i_1^2 = 90 \times (0.2)^2 = 3.6W$$

2.5 电压源和电流源的串联和并联

1. 理想电压源的串联和并联

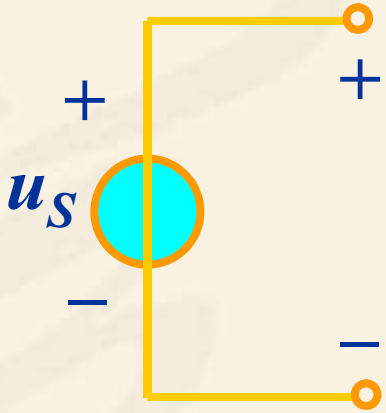
注意参考方向

串联

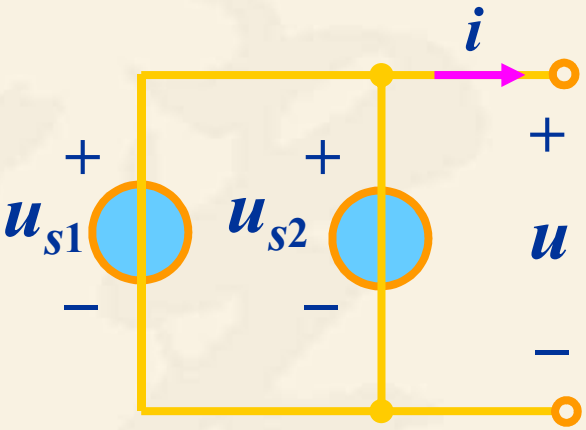


$$u_s = u_{s1} + u_{s2} = \sum u_{sk}$$

等效电路



并联

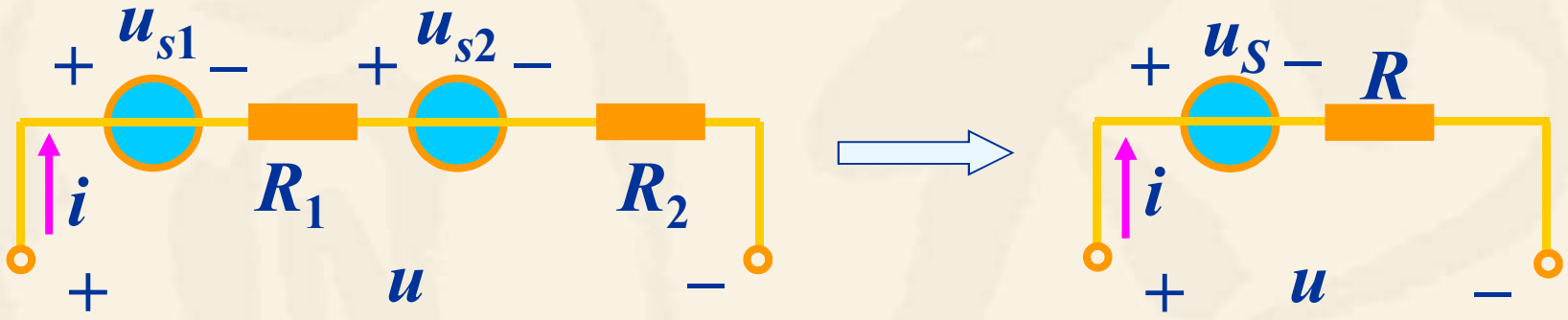


等效电路

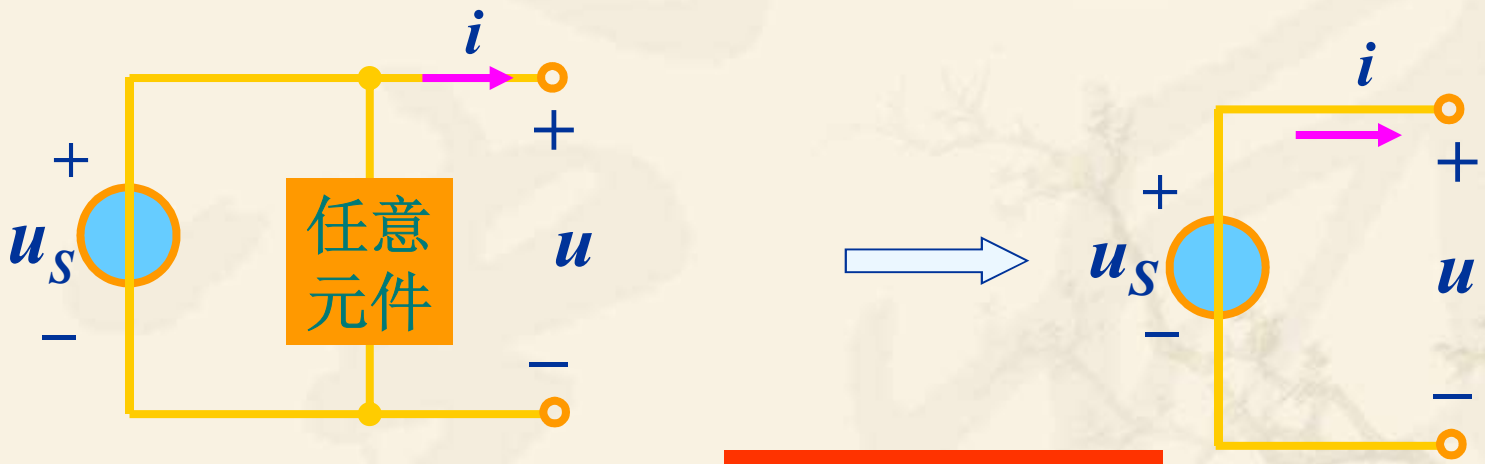
$$u_s = u_{s1} = u_{s2}$$

相同的电压源才能并联，电源中的电流不确定。

电压源与支路的串、并联等效



$$u = u_{S1} + R_1 i + u_{S2} + R_2 i = (u_{S1} + u_{S2}) + (R_1 + R_2) i = u_S + Ri$$



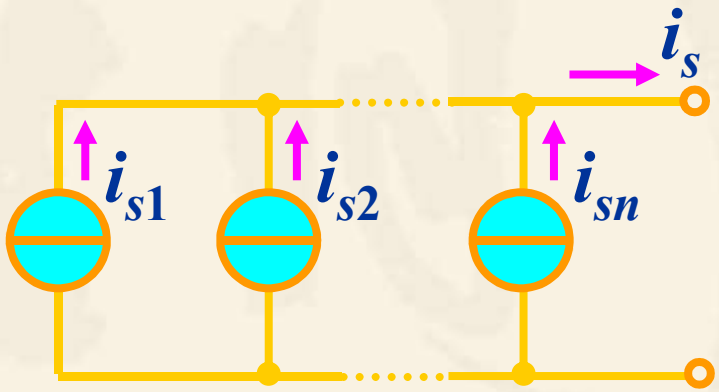
对外等效！

2. 理想电流源的串联并联

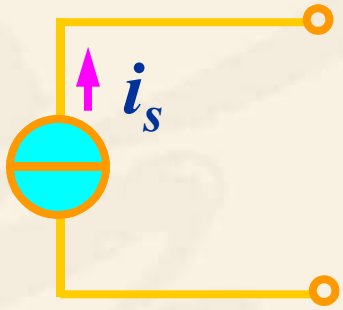
注意参考方向

并联

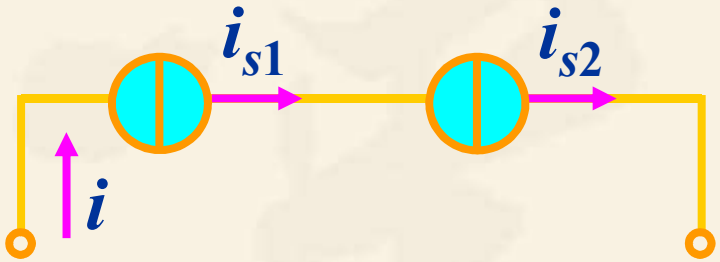
$$i_s = i_{s1} + i_{s2} + \cdots + i_{sn} = \sum i_{sk}$$



等效电路



串联

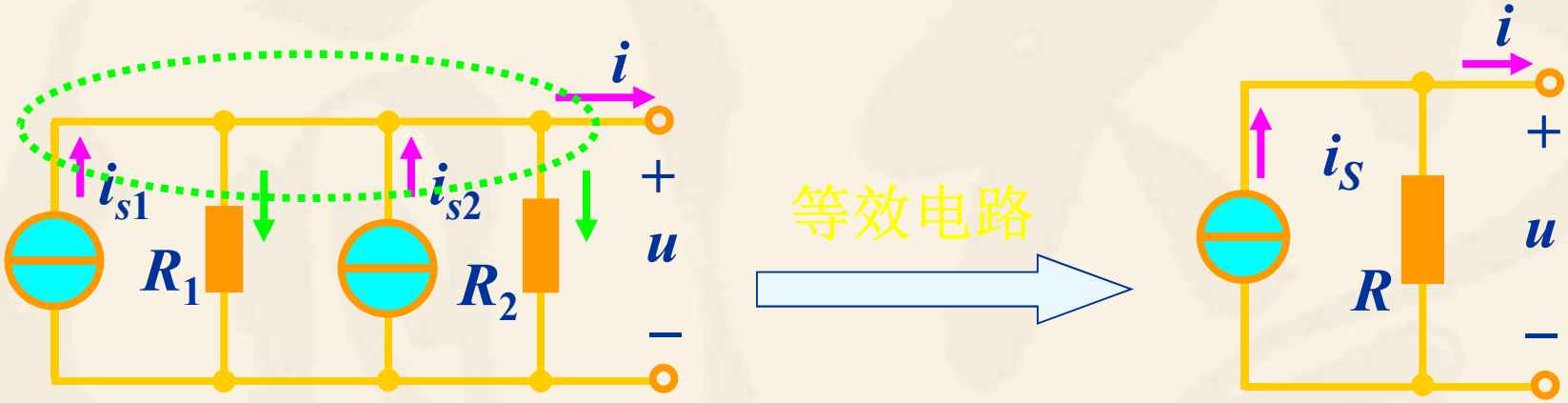


等效电路

$$i_s = i_{s1} = i_{s2}$$

相同的理想电流源才能串联, 每个电流源的端电压不能确定

电流源与支路的串、并联等效

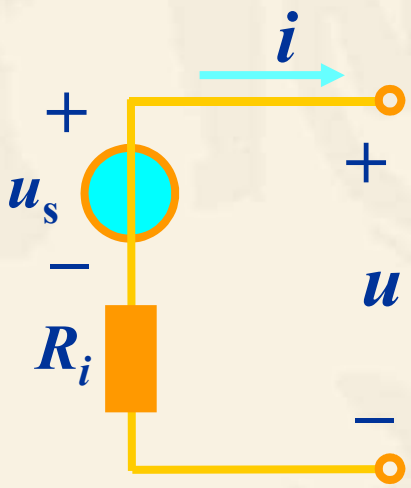


$$i = i_{S1} - u/R_1 + i_{S2} - u/R_2 = (i_{S1} + i_{S2}) - (1/R_1 + 1/R_2)u = i_S - u/R$$

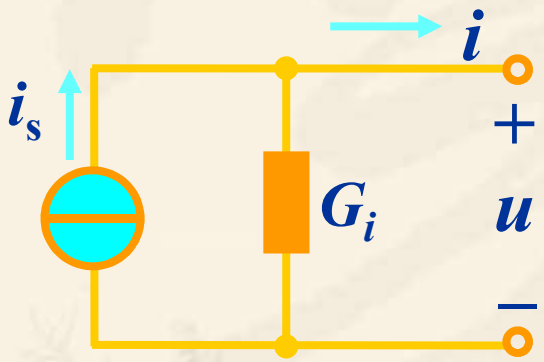
对外等效！

2.6 实际电压源和实际电流源的等效变换

实际电压源、实际电流源两种模型可以进行等效变换，所谓的等效是指端口的电压、电流在转换过程中保持不变。



实际电压源



实际电流源

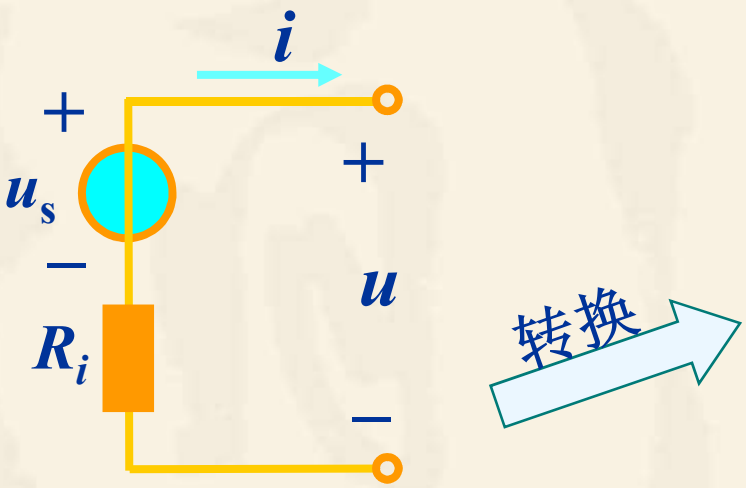
$$u = u_S - R_i\, i$$

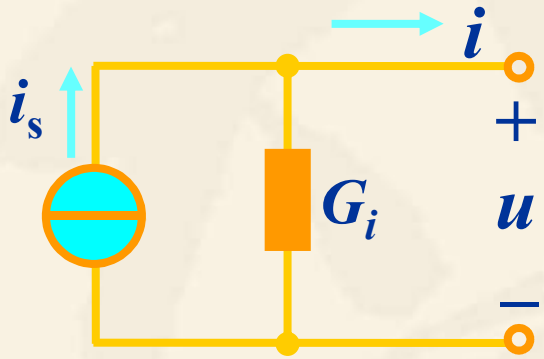
$$i = \frac{u_S}{R_i} - \frac{u}{R_i}$$

$$i = i_S - G_i u$$

可得等效的条件 $i_S = \dfrac{u_S}{R_i}$, $G_i = \dfrac{1}{R_i}$

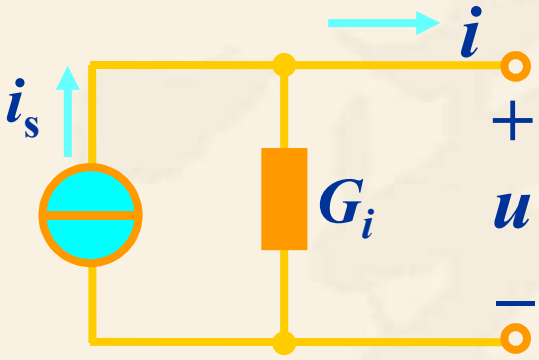
由电压源变换为电流源：

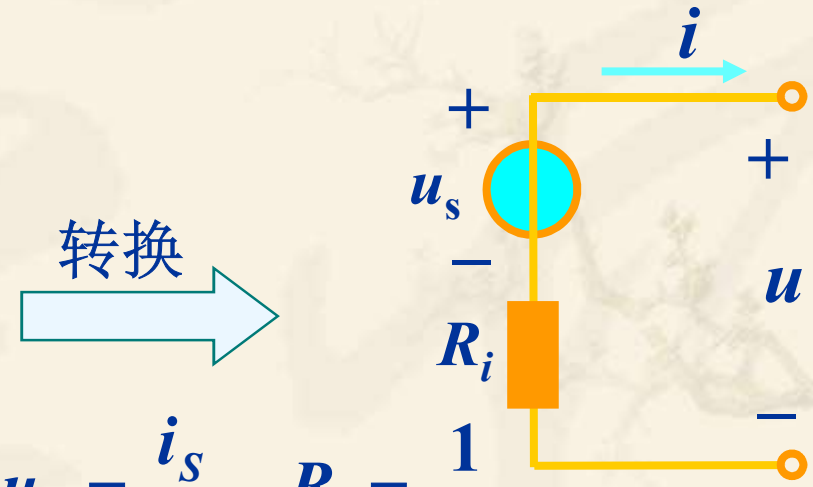




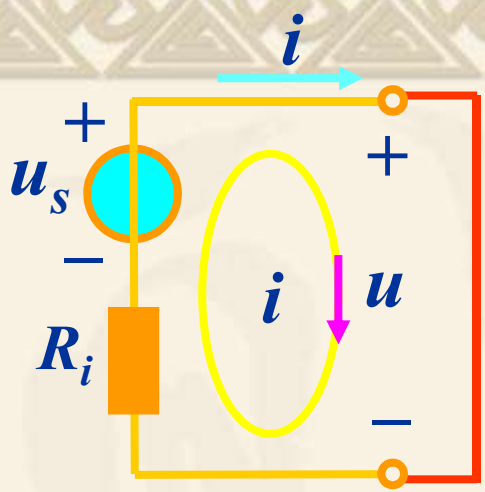
$$i_S = \frac{u_S}{R_i},\quad G_i = \frac{1}{R_i}$$

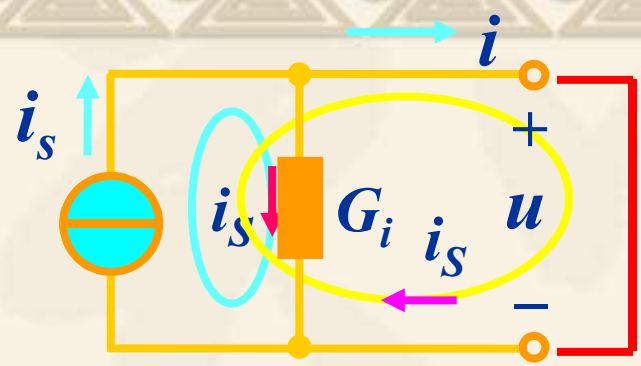
由电流源变换为电压源：





$$u_S = \frac{i_S}{G_i},\quad R_i = \frac{1}{G_i}$$





注意

(1) 变换关系 $\begin{cases} \text{数值关系；} \\ \text{方向：电流源电流方向为电压源电压升方向。} \end{cases}$

(2) 等效是对外部电路等效，对内部电路是不等效的。

表现在

- 开路的电压源中无电流流过 R_i；

  开路的电流源可以有电流流过并联电导 G_i。

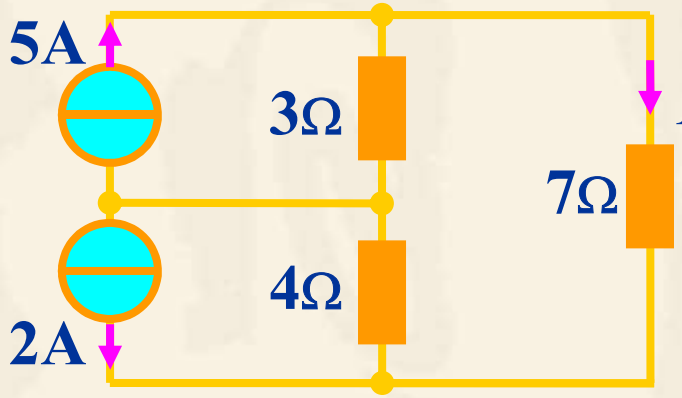
- 电压源短路时，电阻中 R_i 有电流；

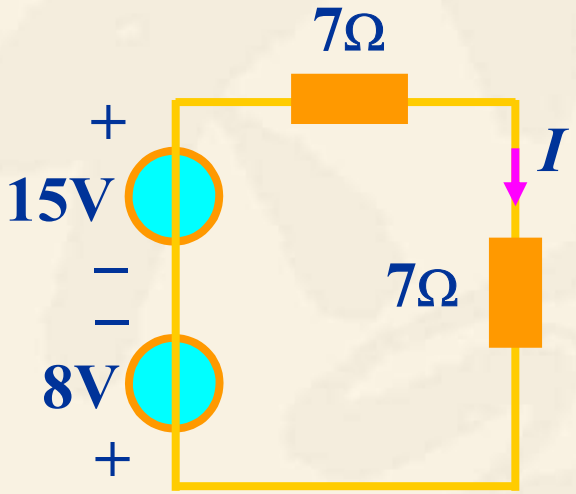
  电流源短路时，并联电导 G_i 中无电流。

(3) 理想电压源与理想电流源不能相互转换。

利用电源转换简化电路计算。

例1

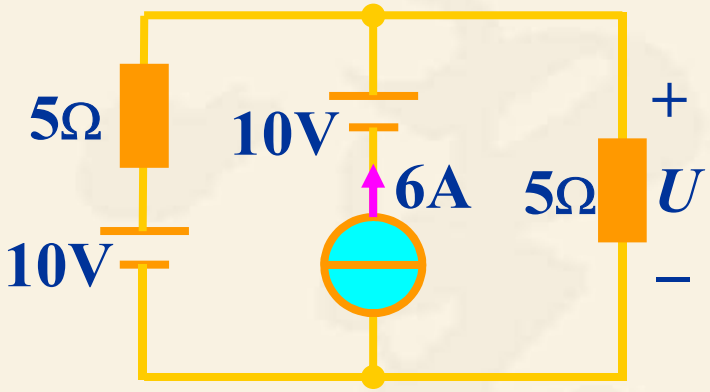


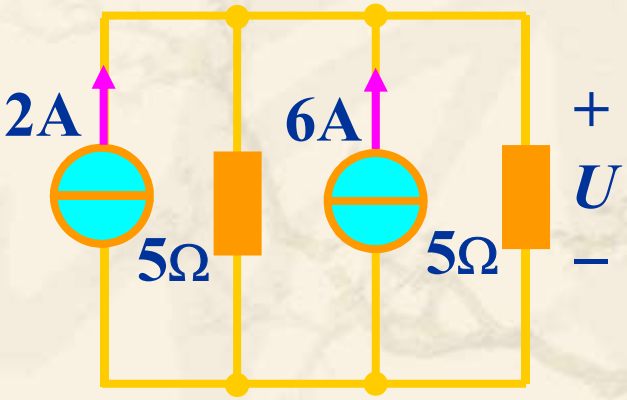


I=0.5A

例2

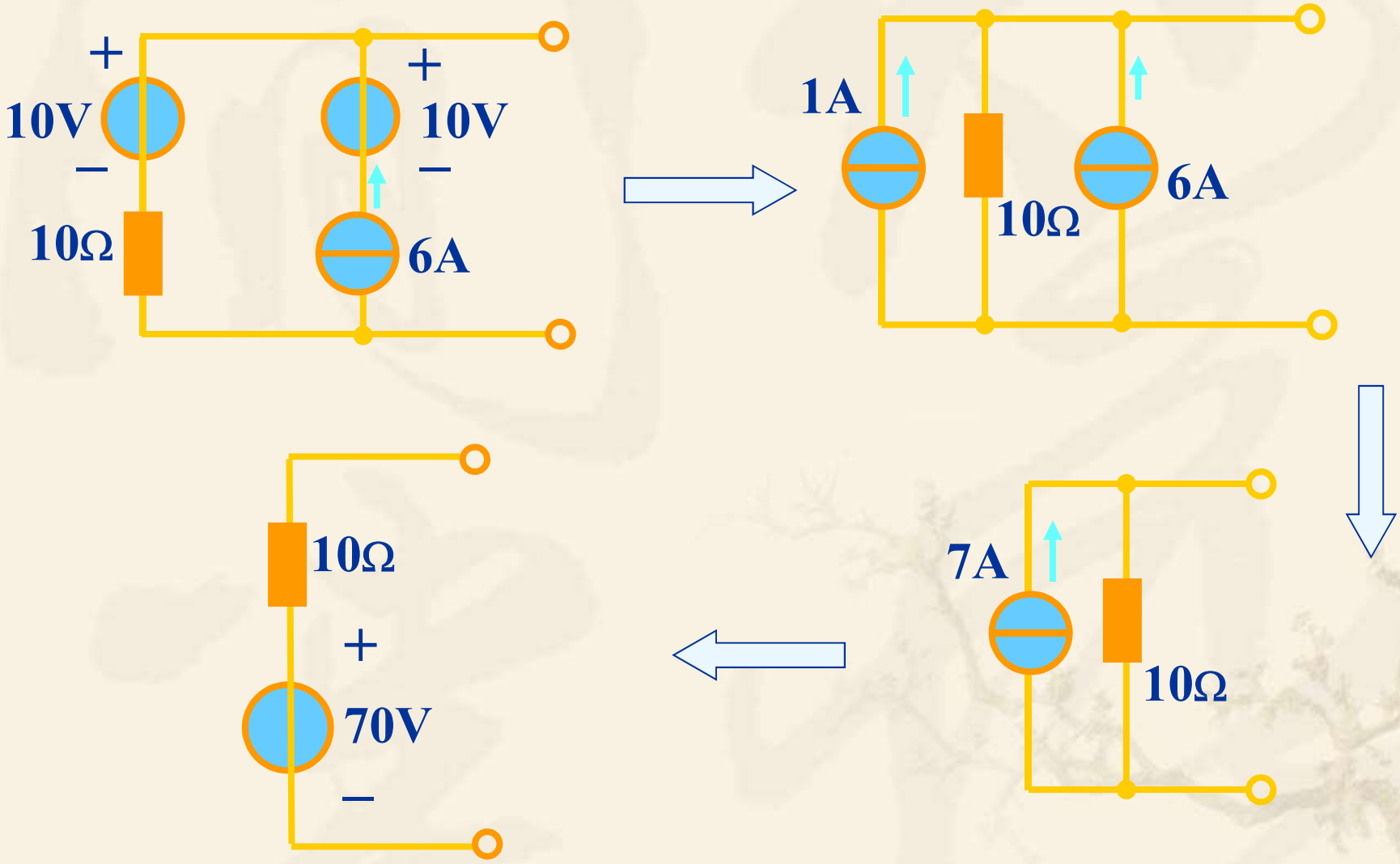
U=?

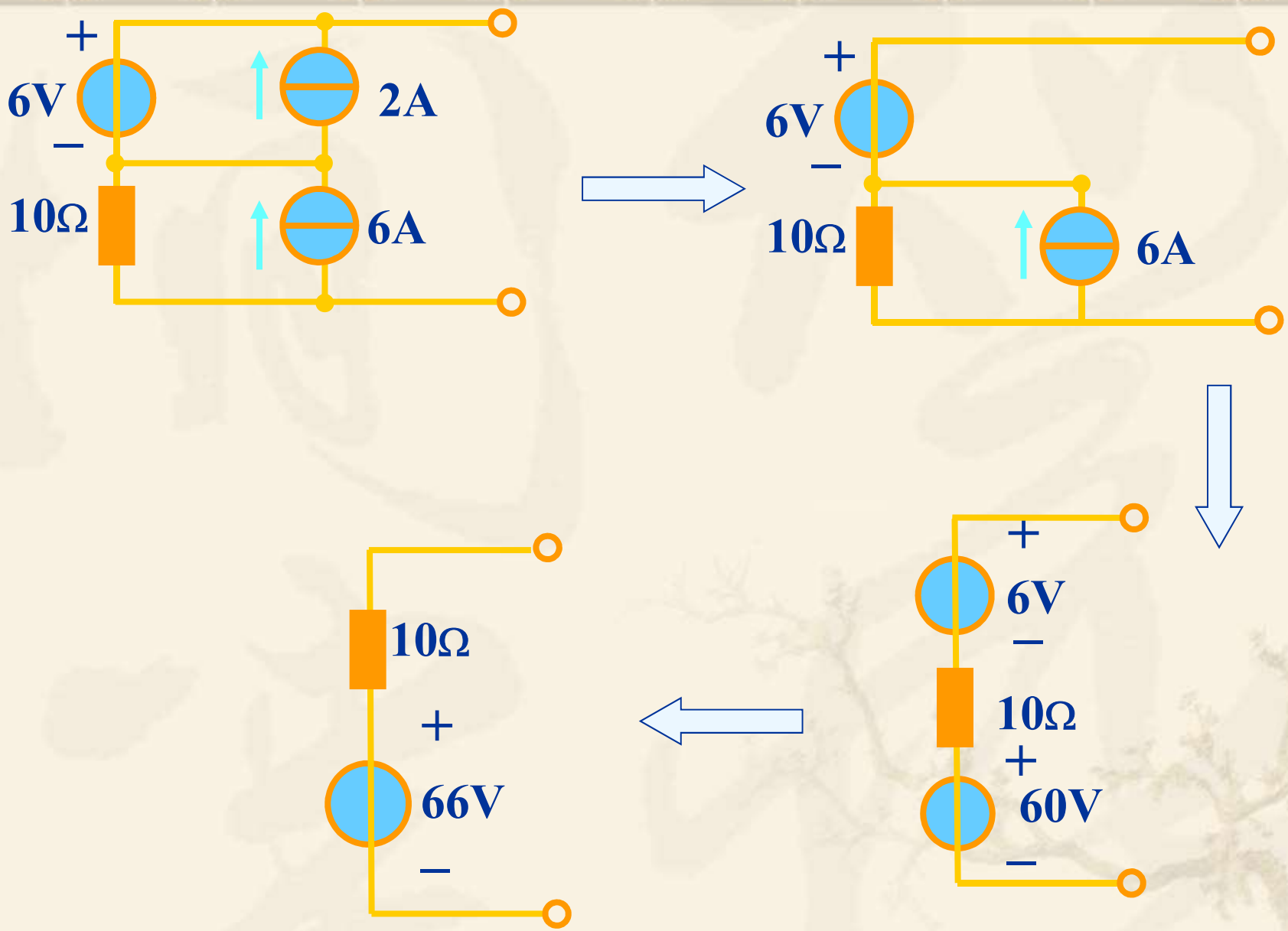




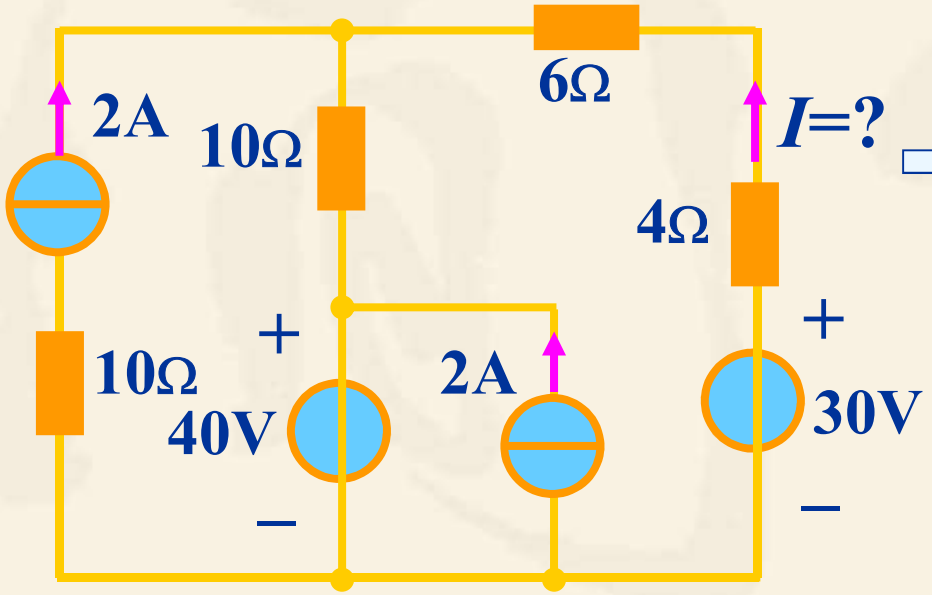
U=20V

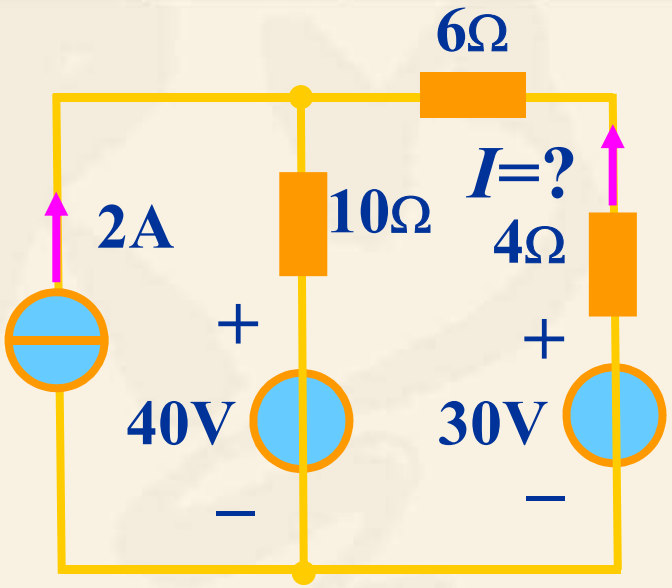
例3 把电路化简成一个电压源和一个电阻的串联。

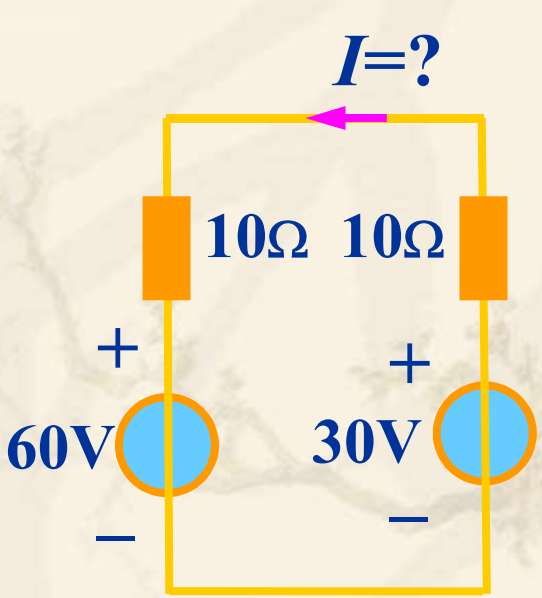



+
6V
−
10Ω
2A
6A
+
6V
−
10Ω
6A
+
6V
−
10Ω
+
60V
−
10Ω
+
66V
−

例4

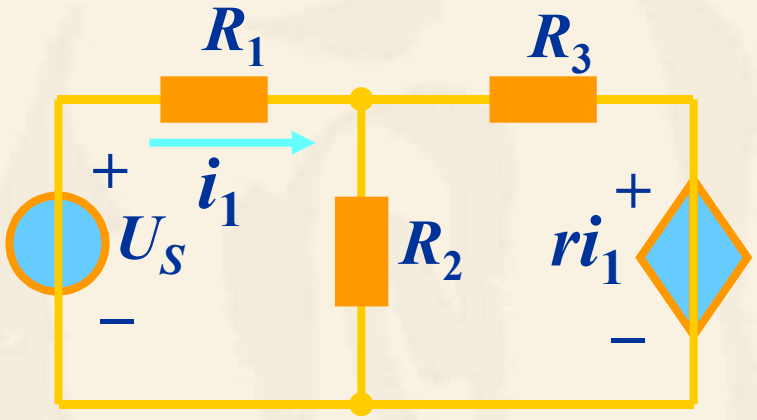


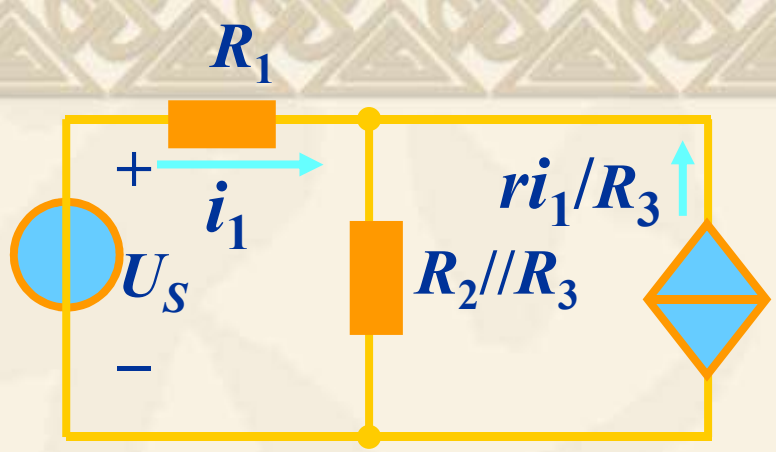


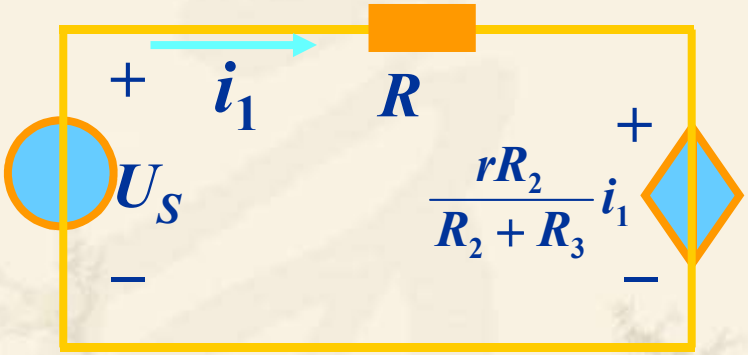


$$I = \frac{30-60}{20} = -1.5A$$

例5 求电流i_1







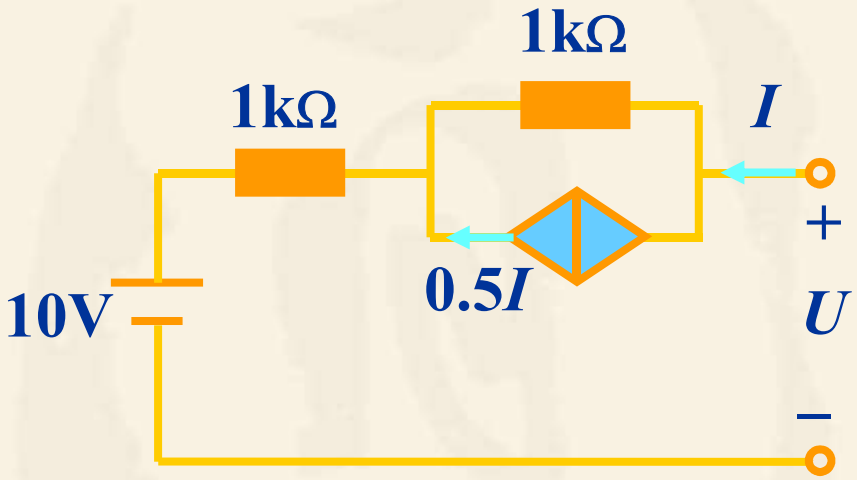
$$R = R_1 + \frac{R_2 R_3}{R_2 + R_3}$$

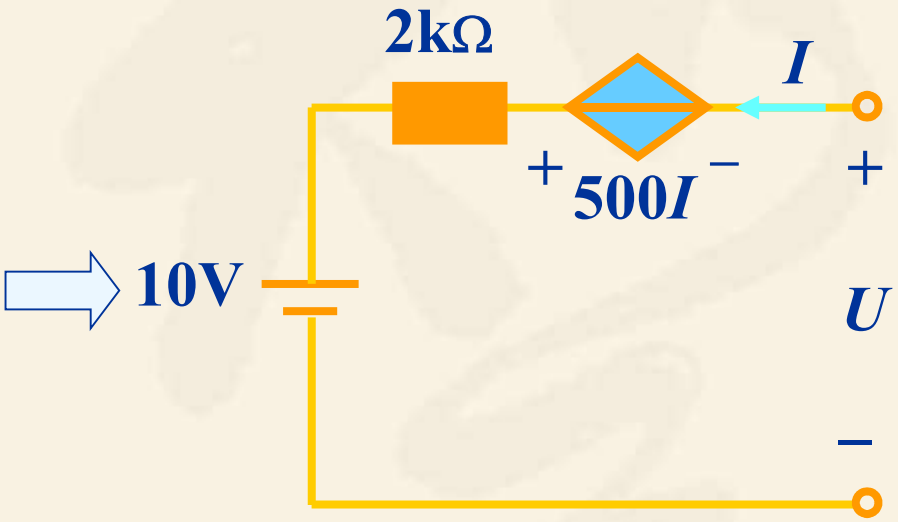
$$Ri_1 + \frac{rR_2}{R_2 + R_3} i_1 = U_S$$

$$i_1 = \frac{(R_2 + R_3)U_S}{R_1R_2 + R_1R_3 + R_2R_3 + rR_2}$$

注意！！！ 受控源和独立源一样可以进行电源转换；转换过程中注意不要丢失控制量。

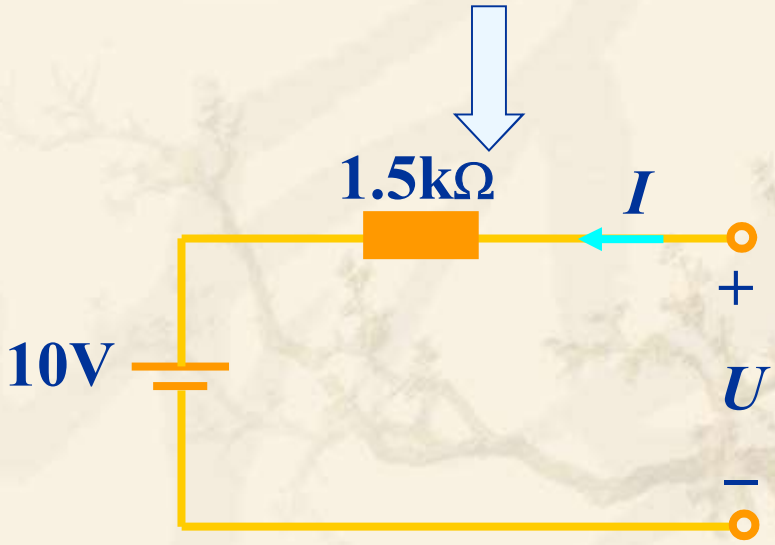
例6 把电路转换成一个电压源和一个电阻的串联。





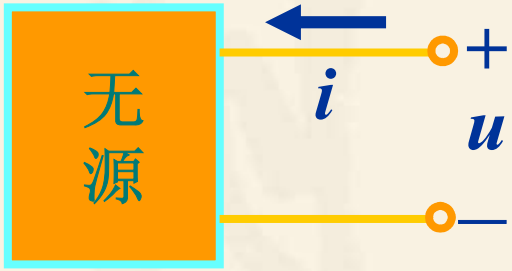
$$U = -500I + 2000I + 10$$

$$= 1500I + 10$$



2.7 输入电阻

1. 定义



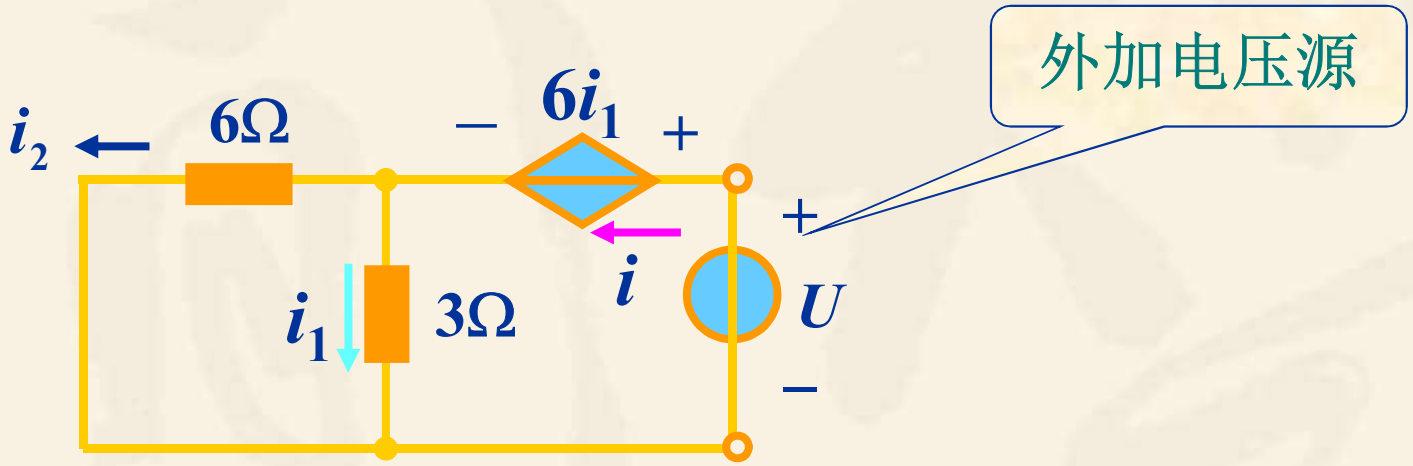
输入电阻 $R_{in} = \dfrac{u}{i}$

2. 计算方法

（1）如果一端口内部仅含电阻，则应用电阻的串、并联和Δ—Y变换等方法求它的等效电阻；

（2）对含有受控源和电阻的两端电路，用电压、电流法求输入电阻，即在端口加电压源，求得电流，或在端口加电流源，求得电压，得其比值。

例2 求一端口的等效输入电阻。

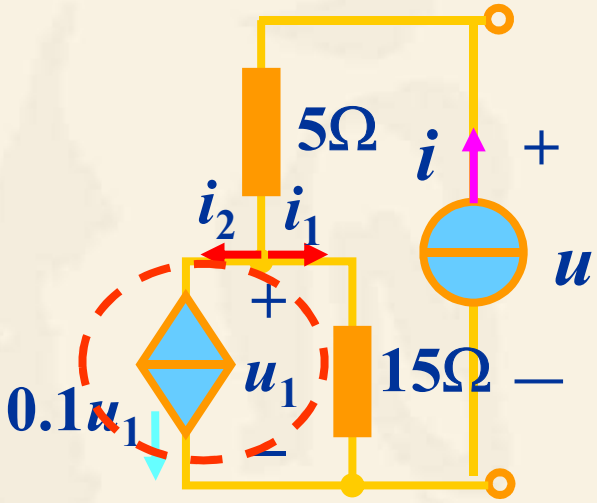


$$i = i_1 + i_2 = i_1 + \frac{3i_1}{6} = 1.5i_1$$

$$U = 6i_1 + 3i_1 = 9i_1$$

$$R_{in} = \frac{U}{i} = \frac{9i_1}{1.5i_1} = 6\Omega$$

例3



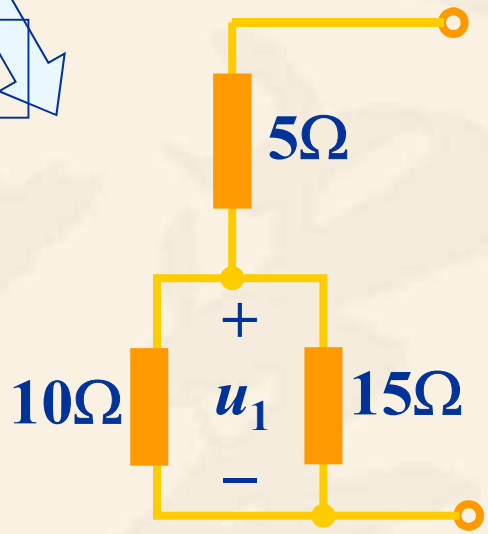
$$u_1 = 15i_1$$

$$i_2 = 0.1u_1 = 1.5i_1$$

$$i = i_1 + i_2 = 2.5i_1$$

$$u = 5i + u_1 = 5\times 2.5i_1 + 15i_1 = 27.5i_1$$

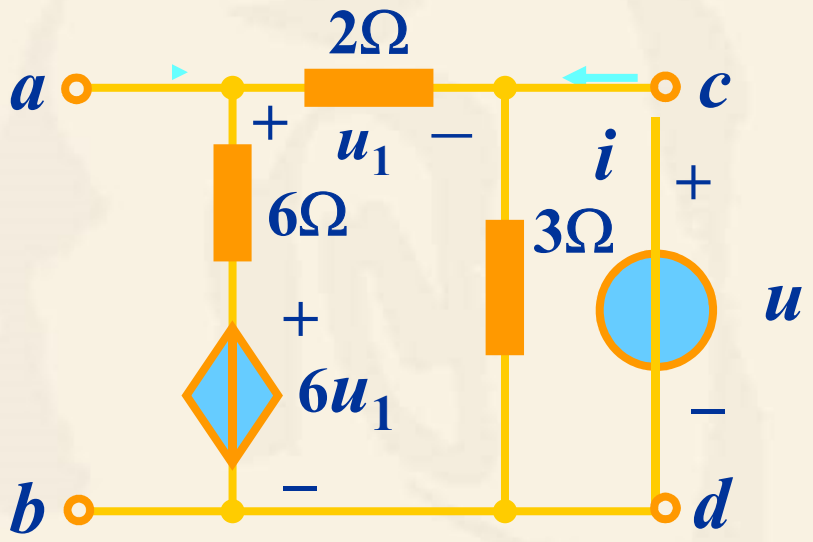
等效



$$R_{in} = \frac{u}{i} = \frac{27.5i_1}{2.5i_1} = 11\Omega$$

$$R_{in} = 5 + \frac{10\times 15}{10+15} = 11\Omega$$

例4 求R_{ab}和R_{cd}



$$u = u_1 + 3u_1/2 = 2.5u_1$$

$$u_1 = u/2.5 = 0.4u$$

$$i = \frac{u_1}{2} + \frac{u - 6u_1}{6} = \frac{-u}{30}$$

$$R_{ab} = \frac{u}{i} = -30\Omega$$

$$u = -u_1 + 6u_1 + 6\times(-\frac{u_1}{2}) = 2u_1$$

$$i = -u_1/2 + u/3 = u_1/6$$

$$R_{cd} = \frac{u}{i} = 12\Omega$$

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